Mirror-Prox SCA Algorithm for Multicast Beamforming and Antenna Selection

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Abstract

- Multicast transmit beamforming - an effective technique for increasing throughput in multi-antenna systems
- In practice, the BS may have more antennas than RF chains
  - ➢ antenna elements - small and inexpensive
  - ➢ RF chains – bulky, expensive, power consuming
- Use limited number of RF chains to perform multicast beamforming?
  - ➢ *Joint multicast beamforming and antenna selection*
- However, the problem is NP-hard
- Prior art: uses semi-definite relaxation (SDR)
  - ➢ high computational complexity and lack of scalability
- We develop a high performance, low complexity algorithm to handle the joint problem

Background

Joint multicast beamforming and antenna selection

Goal: jointly select the "best" subset of antennas and the corresponding beam-forming vectors that can maximize the minimum received SNR among the users

- Problem statement: downlink transmission in a single cell, M-user MISO system served by a BS with N antennas

$$\max_{\mathbf{w}\in\mathbb{C}^n} \min_m \mathbf{w}^H\mathbf{Q}_m\mathbf{w} \quad \text{s. to} \quad \|\mathbf{w}\|^2 \le P, \quad \|\mathbf{w}\|_0 \le K$$

$$\Rightarrow$$

$$\max_{\mathbf{w}\in\mathbb{C}^n} \min_m \mathbf{w}^H\mathbf{Q}_m\mathbf{w} - \lambda\|\mathbf{w}\|_1 \quad \text{s. to} \quad \|\mathbf{w}\|^2 \le P$$

- Non-convex and NP-hard problem
- Use approximation algorithms for computing high quality sub-optimal solutions
- SDR + Gaussian randomization: effective in identifying near-optimal solutions for the problem at high computational cost. [Mehanna'13]
- Alternative: Use *successive convex approximation* (SCA), a general approximation framework for non-convex problems

SCA: Iteratively solve a series of convex problems obtained by constructing a convex surrogate of the non-convex objective function at each iteration

- First-order based methods to solve each convex sub-problem has been used to handle the problem without antenna selection [Konar'17]
  - ➢ High performance at low complexity
  - ➢ Fast convergence to a high quality (approximate) solution
- Saddle-Point Mirror-Prox (SP-MP) algorithm: A first-order primal-dual algorithm for efficiently solving problems of the form

$$\min_{\mathbf{x}\in\mathcal{X}} \max_{\mathbf{y}\in\mathcal{Y}} \phi(\mathbf{x},\mathbf{y})$$

  - ➢ The sets $\mathcal{X}$ and $\mathcal{Y}$ are "simple", convex and compact
  - ➢ $\phi(\mathbf{x},\mathbf{y})$ is convex in **x** and concave in **y**

Motivation

Can we apply the SDR-based technique to massive MIMO scenarios for the joint problem?

- ➢ SDR lifts the problem to higher dimensional space
- ➢ Computational complexity of solving SDP: O(N^6.5), so what if N↑?
- This motivates the search for an approach that :
  - ➢ returns high quality approximate solutions
  - ➢ applicable to massive MIMO scenarios ➡ **SP-MP SCA**
  - ➢ more computationally efficient than SDR

Q: Why use SP-MP to solve each convex SCA sub-problem?
A: Features dimension-independent O(1/t) -rate of convergence [Nemirovski' 04] with cheap (O(MN)) per-iteration costs

Problem formulation

- Consider the joint problem in the real domain

$$\max_{\bar{\mathbf{w}}\in\mathbb{R}^{2N}} \min_{m\in[M]} \bar{\mathbf{w}}^H\tilde{\mathbf{Q}}_m\bar{\mathbf{w}} - \lambda\|\bar{\mathbf{w}}\|_1 \quad \text{s.t.} \quad \|\bar{\mathbf{w}}\|_2^2 \le P$$

- By applying the following equality to the above problem

$$\max_{\mathbf{x}\in\mathcal{X}} \min_{m\in[M]} f_m(\mathbf{x}) \Leftrightarrow \min_{\mathbf{x}\in\mathcal{X}} \max_{m\in[M]} -f_m(\mathbf{x})$$

$$\Downarrow$$

$$\min_{\bar{\mathbf{w}}\in\mathbb{R}^{2N}} \max_{m\in[M]} \bar{\mathbf{w}}^H\tilde{\mathbf{Q}}_m\bar{\mathbf{w}} + \lambda\|\bar{\mathbf{w}}\|_1 \quad \text{s.t.} \quad \|\bar{\mathbf{w}}\|_2^2 \le P$$

- Group sparsity and Dual norm:

$$\min_{\bar{\mathbf{w}}\in\mathbb{R}^{2N}} \max_{m\in[M]} \bar{\mathbf{w}}^H\tilde{\mathbf{Q}}_m\bar{\mathbf{w}} + \lambda\|\bar{\mathbf{w}}\|_{1,2} \quad \text{s.t.} \quad \|\bar{\mathbf{w}}\|_2^2 \le P$$

$$\min_{\bar{\mathbf{w}}\in\mathbb{R}^{2N}} \max_{m\in[M]} \bar{\mathbf{w}}^H\tilde{\mathbf{Q}}_m\bar{\mathbf{w}} + \lambda \max_{\|\mathbf{s}\|_{\infty,2}\le 1} \mathbf{s}^T\bar{\mathbf{w}} \quad \text{s.t.} \quad \|\bar{\mathbf{w}}\|_2^2 \le P$$

- Applying the SCA approach

$$\min_{\bar{\mathbf{w}}\in\mathbb{R}^{2N}} \max_{m\in[M]} \mathbf{a}_m^{(n)T}\bar{\mathbf{w}} + b_m^{(n)} + \lambda \max_{\|\mathbf{s}\|_{\infty,2}\le 1} \mathbf{s}^T\bar{\mathbf{w}} \quad \text{s.t.} \quad \|\bar{\mathbf{w}}\|_2^2 \le P$$

- Expressing the piece-wise linear function as a linear maximization problem over the simplex

$$\min_{\bar{\mathbf{w}}\in\mathbb{R}^{2N}} \max_{\mathbf{y}\in\triangle_M} \mathbf{y}^T(\mathbf{A}_m^{(n)}\bar{\mathbf{w}} + \mathbf{b}_m^{(n)}) + \lambda \max_{\|\mathbf{s}\|_{\infty,2}\le 1} \mathbf{s}^T\bar{\mathbf{w}} \quad \text{s.t.} \quad \|\bar{\mathbf{w}}\|_2^2 \le P$$

$$\min_{\bar{\mathbf{w}}\in\mathcal{W}} \max_{\bar{\mathbf{x}}\in\triangle_M\times\mathcal{S}} \bar{\mathbf{x}}^T(\bar{\mathbf{A}}_m^{(n)}\bar{\mathbf{w}} + \bar{\mathbf{b}}^{(n)})$$

$$\Leftarrow$$

$$\min_{\bar{\mathbf{w}}\in\mathcal{W}} \max_{\bar{\mathbf{x}}\in\mathcal{X}} \phi^{(n)}(\bar{\mathbf{w}},\bar{\mathbf{x}})$$

SP-MP for SCA

- Note that $\phi^{(n)}(.,.)$ is bilinear and $\mathcal{W}$ and $\mathcal{X}$ are both simple, convex and compact set
- By Sion's Minimax theorem

$$\min_{\bar{\mathbf{w}}\in\mathcal{W}} \max_{\bar{\mathbf{x}}\in\mathcal{X}} \phi^{(n)}(\bar{\mathbf{w}},\bar{\mathbf{x}}) = \min_{\bar{\mathbf{x}}\in\mathcal{X}} \max_{\bar{\mathbf{w}}\in\mathcal{W}} \phi^{(n)}(\bar{\mathbf{w}},\bar{\mathbf{x}})$$

- The optimal solution pair is a *saddle point* that can be efficiently computed using SP-MP ☺
- The problem is ready to be solved by the SP-MP SCA algorithm
  - ➢ SP-MP is a variant of the mirror descent algorithm
  - ➢ SP-MP uses non-Euclidean projections (computed using Bregman divergences)
- The projection on the three sets $\mathcal{W}$, $\mathcal{S}$ and $\triangle_M$ can be simply performed (closed form solutions)
  - ➢ $\mathcal{S}$ ➡ project each sub-vector onto unit l2 ball w.r.t. standard Euclidean metric
  - ➢ $\triangle_M$ ➡ project onto M-dimensional probability simplex w.r.t. (un-normalized) KL-divergence

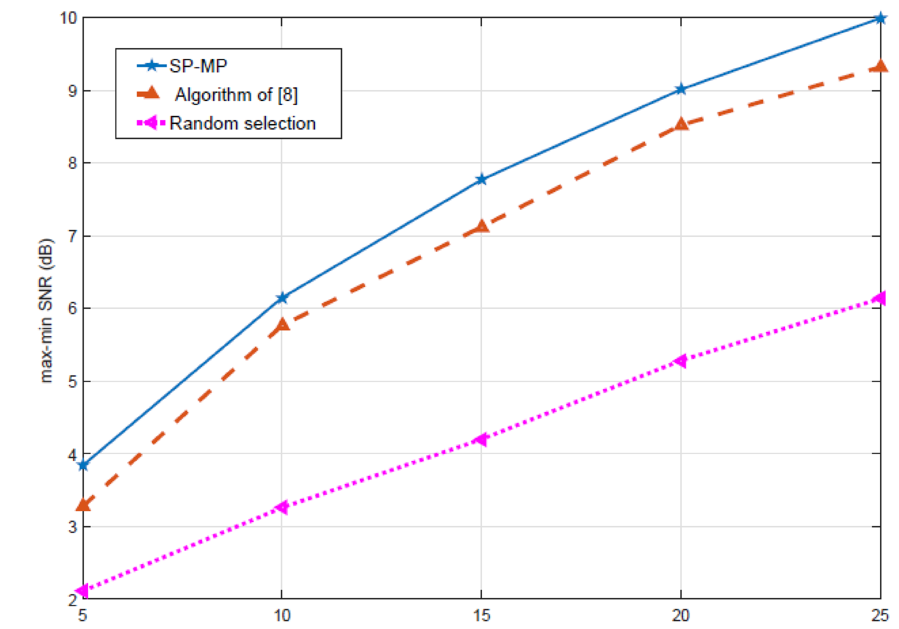
Experiments

- Compare the quality and timing performance of the SP-MP SCA algorithm vs the SDR approach
- The downlink channel is generated as

$$\mathbf{h}_m^H = \sqrt{\frac{N}{L_m}} \sum_{l=1}^{L_m} \alpha_m^{(l)} \mathbf{a}_t(\theta^{(l)})^H, \forall m = 1, \cdots, M$$

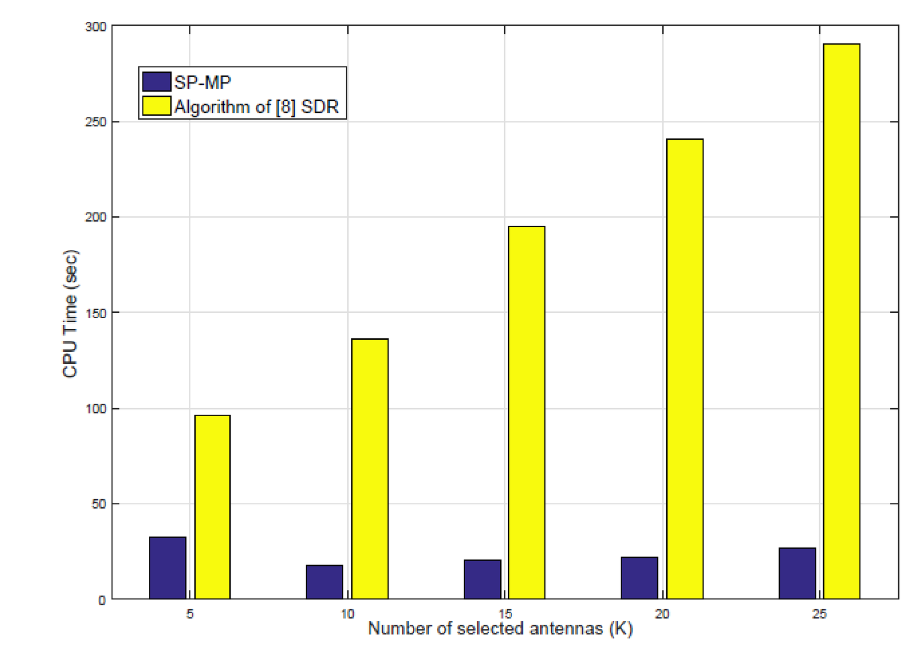
Simulation parameters :

- Consider a scenario with N = 30 and M = 50
- P = 10 dB, noise variance = 1
- Outer loop iterations is equal to 10, and 1000 iterations are used for each sub-problem
- Test for different K (number of selected antennas)
- For each K, 200 Monte-Carlo trials
- For binary search, $\lambda_{UB} = 2$ and $\lambda_{LB} = 0$

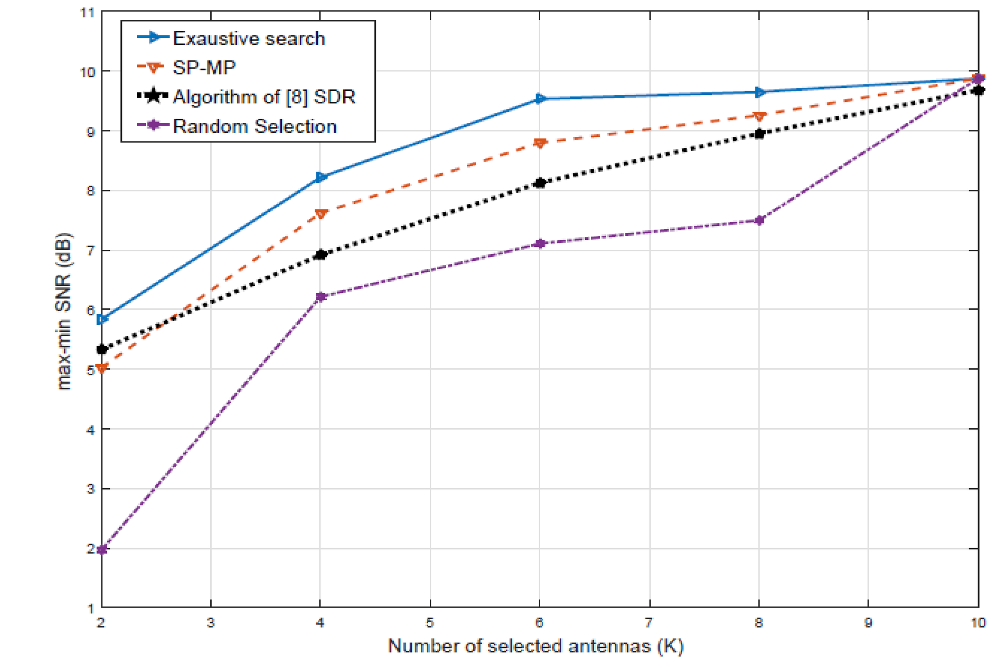


max-min SNR vs K, N=30

Experiments (cont.)



- Consider scenario with N = 10 and M = 16
  - ➢ Affordable to run exhaustive search (upper bound)
  - ➢ Allows us to evaluate quality of solution returned by SP-MP SCA



Results and Conclusion

- Proposed a first order-based method to handle the joint multicast beamforming and antenna selection problem, which can be applied for massive MIMO scenarios
- Key idea: Using SCA approach followed by a fast converging SP-MP algorithm to obtain high quality, low-complexity solution
- Flexibility: The algorithm can be easily applied to the case where N is greater than M, and per antenna power constraint instead of the sum power one.