A FAST APPROXIMATION ALGORITHM FOR SINGLE-GROUP MULTICAST BEAMFORMING WITH LARGE ANTENNA ARRAYS

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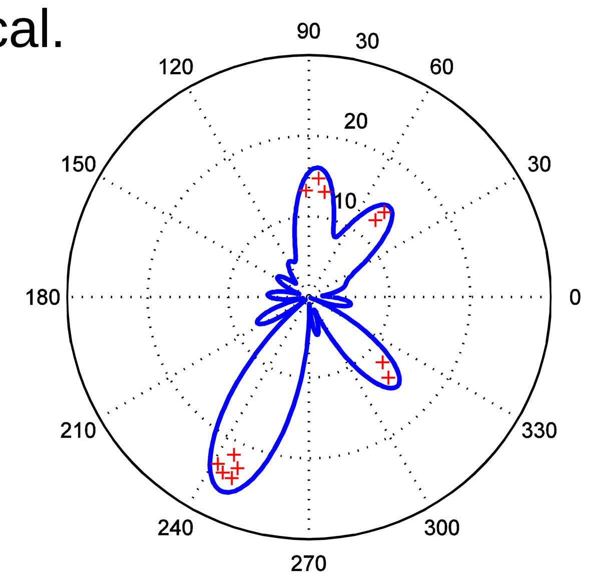
Introduction

- **Multicast beamforming** – Uses multiple transmit antennas to send the same data (e.g., video) stream to a group of users @ given QoS.
- **Channel State Information at the Transmitter (CSIT)** – required; can be instantaneous, long-term, or statistical.
- **Maximum multicast rate** determined by weakest link (smallest SNR).
- **Multicast beamformer design** formulated as an optimization problem
  - i) Max-Min (SNR) under power constraint
  - ii) Min power under min SNR constraints
- **Non-convex and NP-hard**



Related Work

- **SDR-G** – [Sidiropoulos *et al.*, 2006]. Semi-Definite Relaxation (SDR) followed by Gaussian randomization.
- **Successive Linear Approximation (SLA)** – [Tran *et al.*, 2014] Iteratively solve sequence of QP problems.
- **Alternating Maximization (AM)** – [Demir *et al.*, 2014] Iteratively solve sequence of SDP problems; high complexity.
- **Lozano's algorithm** – [Lozano, 2007] Step in the direction of the SNR gradient of the bottleneck user, then scale. May not converge; limit cycles [Matskani *et al.*, 2009].
- **Damped Lozano's algorithm with Lopez initialization (dLLI)** – [Matskani *et al.*, 2009].
- **QR algorithm** – [Abdelkader *et al.*, 2010] Uses channel orthogonalization based on QR decomposition.
- **Multiplicative (MU) Algorithm** – [Gopalakrishnan *et al.*, 2015] Uses proportional-fairness as a surrogate for max-min fairness, takes into account SNR gradient of all users, achieves best performance-complexity tradeoff.

System Model

- Single-group multicast cell, single Transmitter (Tx) with N antennas serving M single receive antenna users. **Perfect CSIT assumed.**
- Received signal at user m: $y_m = \mathbf{h}_m^H \mathbf{w} x + z_m,\ \forall\, m \in \mathcal{M} = \{1, \cdots, M\}$
- SNR at user m : $\frac{|\mathbf{h}_m^H \mathbf{w}|^2}{\sigma_m^2} = \mathbf{w}^H \tilde{\mathbf{R}}_m \mathbf{w},\ \forall\, m \in \mathcal{M}$

Problem Statement

- Max-min fair beamformer design problem

$$\boxed{\max_{\mathbf{w} \in \mathbb{C}^N} \min_{m \in \mathcal{M}} \mathbf{w}^H \tilde{\mathbf{R}}_m \mathbf{w} \ \ \text{s.t.} \ \ \mathbf{w} \in \mathcal{F}}$$

- Constraints can be either sum power or per-antenna
- **Goal:** Design first-order based SLA algorithm for max-min fairness

Problem Reformulation

- **Convert** problem to real domain and then equivalently reformulate as **min-max** problem.

$$\begin{aligned} \max_{\mathbf{w} \in \mathbb{C}^N} \min_{m \in \mathcal{M}} \mathbf{w}^H \tilde{\mathbf{R}}_m \mathbf{w} \ \ \text{s.t.} \ \ \mathbf{w} \in \mathcal{F} \ &\Leftrightarrow \max_{\bar{\mathbf{w}} \in \mathbb{R}^{2N}} \min_{m \in \mathcal{M}} \bar{\mathbf{w}}^T \bar{\mathbf{R}}_m \bar{\mathbf{w}} \ \ \text{s.t.} \ \ \bar{\mathbf{w}} \in \bar{\mathcal{F}} \\ &\Leftrightarrow \min_{\bar{\mathbf{w}} \in \mathbb{R}^{2N}} \max_{m \in \mathcal{M}} \bar{\mathbf{w}}^T \bar{\mathbf{S}}_m \bar{\mathbf{w}} \ \ \text{s.t.} \ \ \bar{\mathbf{w}} \in \bar{\mathcal{F}} \end{aligned}$$

where $\bar{\mathbf{w}} := [\mathbf{w}_r^T, \mathbf{w}_i^T]^T \in \mathbb{R}^{2N},\ \mathbf{w}_r = \mathrm{Re}\{\mathbf{w}\}, \mathbf{w}_i = \mathrm{Im}\{\mathbf{w}\}$

$$\bar{\mathbf{R}}_m := \begin{bmatrix} \mathrm{Re}\{\tilde{\mathbf{R}}_m\} & -\mathrm{Im}\{\tilde{\mathbf{R}}_m\} \\ \mathrm{Im}\{\tilde{\mathbf{R}}_m\} & \mathrm{Re}\{\tilde{\mathbf{R}}_m\} \end{bmatrix} \in \mathbb{R}^{2N \times 2N},\ \bar{\mathbf{S}}_m := -\bar{\mathbf{R}}_m,\ \forall\, m \in \mathcal{M}$$

SLA for max-min fairness

- Define $u_m(\bar{\mathbf{w}}) := \bar{\mathbf{w}}^T \bar{\mathbf{S}}_m \bar{\mathbf{w}},\ \forall\, m \in \mathcal{M}$
- **Linearization** about $\bar{\mathbf{w}} = \bar{\mathbf{w}}^{(k)}$ yields

$$u_m(\bar{\mathbf{w}}) \leq u_m(\bar{\mathbf{w}}^{(k)}) + \nabla u_m(\bar{\mathbf{w}}^{(k)})^T (\bar{\mathbf{w}} - \bar{\mathbf{w}}^{(k)}) = \mathbf{a}_m^{(k)T} \bar{\mathbf{w}} + b_m^{(k)}, \forall\, m \in \mathcal{M}$$

- Furthermore, $\max_{m \in \mathcal{M}} \mathbf{a}_m^{(k)T} \bar{\mathbf{w}} + b_m^{(k)} \geq \max_{m \in \mathcal{M}} u_m(\bar{\mathbf{w}})$
- **Algorithm:**

$$\boxed{\bar{\mathbf{w}}^{(k+1)} = \arg \min_{\bar{\mathbf{w}} \in \bar{\mathcal{F}}} \max_{m \in \mathcal{M}} \mathbf{a}_m^{(k)T} \bar{\mathbf{w}} + b_m^{(k)}}$$

Generates sequence of iterates with monotonically non-increasing cost.

Solving SLA sub-problem

- **Interior point methods:** standard approach, high per-iteration complexity, fast convergence.
- **Subgradient methods:** low per-iteration complexity, slow convergence.
- **Linearized Alternating Direction Methods of Multipliers (L-ADMM):** variant of classical ADMM algorithm, low per-iteration complexity, convergence order of magnitude faster compared to subgradient methods.

L-ADMM

- Sub-problem reformulation

$$\boxed{\min_{\bar{\mathbf{w}} \in \mathbb{R}^{2N}, \bar{\mathbf{z}} \in \mathbb{R}^M} v(\bar{\mathbf{z}}, \bar{\mathbf{w}}^{(k)}) + I_{\bar{\mathcal{F}}}(\bar{\mathbf{w}}) \ \ \text{s.t.} \ \ \mathbf{A}^{(k)} \bar{\mathbf{w}} - \bar{\mathbf{z}} = \mathbf{0}}$$

where $v(\bar{\mathbf{z}}, \bar{\mathbf{w}}^{(k)}) := \max_{m \in \mathcal{M}} \bar{z}_m + b_m^{(k)},\ I_{\bar{\mathcal{F}}}(\bar{\mathbf{w}}) :=$ indicator function of $\bar{\mathcal{F}}$

$$\mathbf{A}^{(k)} := [\mathbf{a}_1^{(k)}, \cdots, \mathbf{a}_M^{(k)}]^T \in \mathbb{R}^{M \times 2N},\ \mathbf{b}^{(k)} := [b_1^{(k)}, \cdots, b_M^{(k)}]^T \in \mathbb{R}^M,$$

- **Update of $\bar{\mathbf{z}}$:** Prox operator of $v(\bar{\mathbf{z}}, \bar{\mathbf{w}}^{(k)})$ (solve via bisection)
- **Update of $\bar{\mathbf{w}}$:** Projection onto $\bar{\mathcal{F}}$ (closed-form)
- Use warm starts to initialize variables in each SLA iteration.

Other approaches

- **Nesterov Smoothing** – [Nesterov, 2004]
- **Mirror-Prox Algorithm** – [Nemirovski, 2004]

Simulation Results

Fig. 1: Avg. min. SNR and Avg. execution time vs. N for M = 50

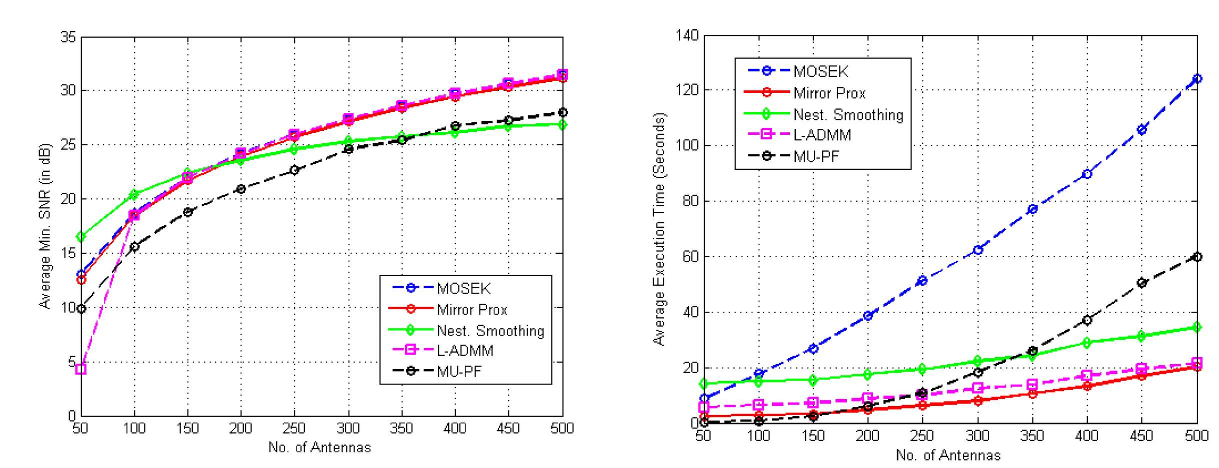
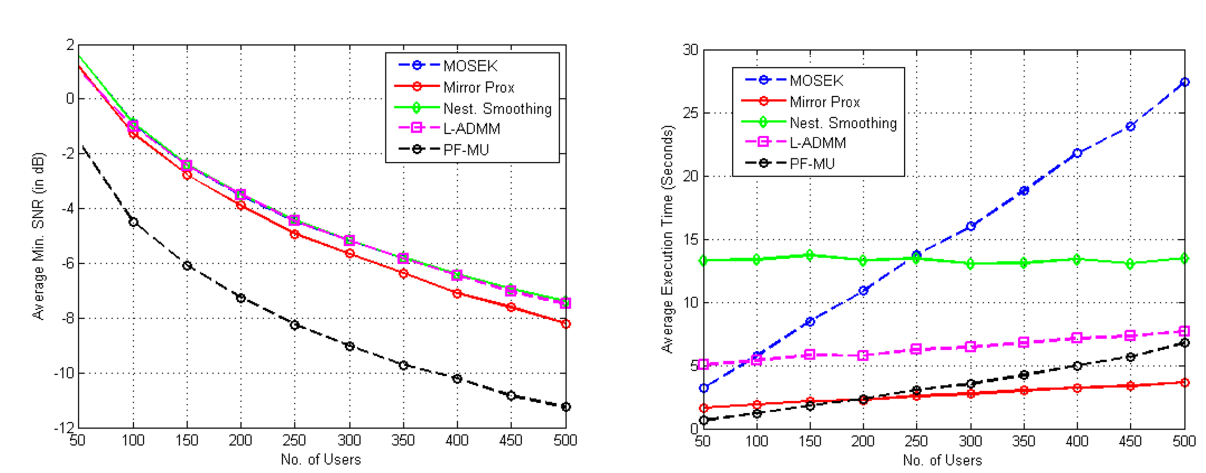


Fig. 2: Avg. min. SNR and Avg. execution time vs. M for N = 25



Conclusions:

- Proposed methods achieve very favorable performance-complexity tradeoff relative to existing state-of-art.
- Future work – Extension to hybrid beamforming architectures for multicasting.