Parametric Frugal Sensing of Autoregressive Power Spectra

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Collaborative Autoregressive Power Spectrum Estimation

- Crowdsourced spectrum sensing: Exploit spatial diversity in distributed sensors to avoid hidden terminal problem, mitigate fading, and enhance spectrum estimation reliability
- Classical spectrum estimation techniques not suitable in distributed sensing scenarios
- Sending analog or finely quantized signal samples creates a heavy burden in terms of communication overhead and battery lifetime ⟹ send one (few) bits/sensor: Frugal Sensing [Mehanna *et al.*, 2013]
- Assumption: signal of interest can be represented by an Autoregressive (AR) model.

Challenge: *collaborative AR spectrum estimation using low-end sensors with limited communication capabilities*

Problem Formulation

Constraints

- Assume single threshold: $t_m = t$
- Received bits $\{b_m\}_{m=1}^M$: $b_m(\mathbf{q}_m^T\mathbf{r}_x - t_m) \geq 0, \forall m \in \{1, \cdots, M\}$
- Spectral non-negativity: $\mathbf{F}\mathbf{r}_x \geq \mathbf{0}$ (approx., but useful for small M)
- Bounded Polyhedron $\mathcal{P}$: (max. power + Cauchy-Schwartz)

Cost function

- Overdetermined YW fitting criterion: $\|\tilde{\mathbf{R}}_x\tilde{\boldsymbol{\beta}} - \mathbf{e}_1^K\|_2^2$

$$\tilde{\mathbf{R}}_x\tilde{\boldsymbol{\beta}} - \mathbf{e}_1^K = \begin{bmatrix} r_x(0) & r_x(-1) & \cdots & r_x(-p) \\ r_x(1) & r_x(0) & \cdots & r_x(-p+1) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(p) & r_x(p-1) & \cdots & r_x(0) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(K-1) & r_x(K-2) & \cdots & r_x(K-p-1) \end{bmatrix} \begin{bmatrix} 1 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

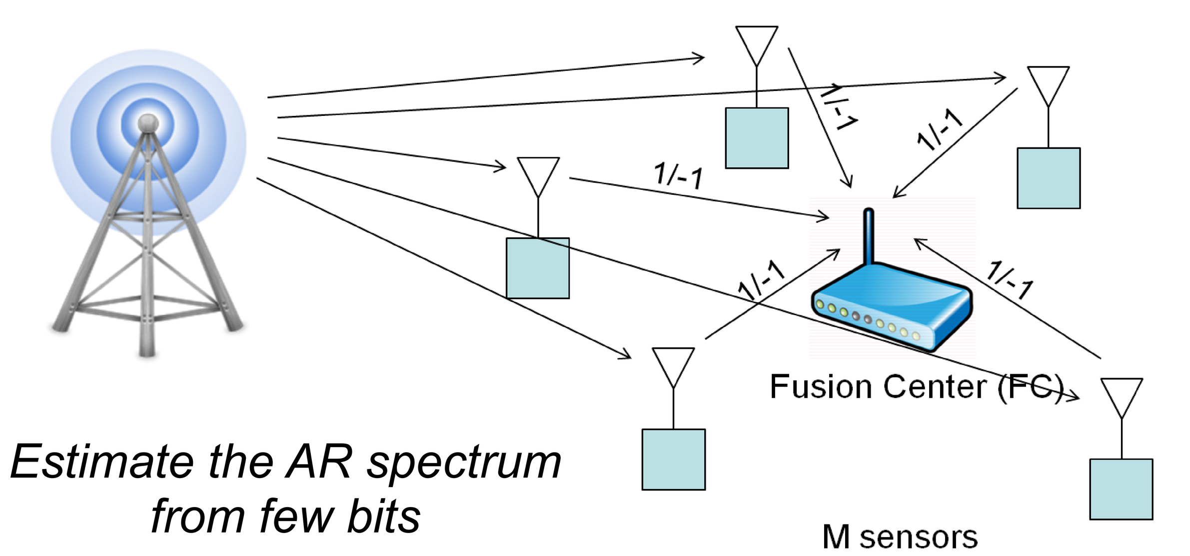
Problem Formulation

$$\begin{aligned} \min_{\mathbf{r}_x, \boldsymbol{\beta}} \quad & \|\tilde{\mathbf{R}}_x\tilde{\boldsymbol{\beta}} - \mathbf{e}_1^K\|_2^2 \\ \text{s.t.} \quad & b_m(\mathbf{q}_m^T\mathbf{r}_x - t_m) \geq 0, \ \forall\, m \in \{1, \cdots, M\} \\ & \mathbf{F}\mathbf{r}_x \geq \mathbf{0}, \\ & \mathbf{r}_x \in \mathcal{P} \end{aligned}$$

Non-convex problem

AR Power Spectrum: $S_x(e^{j\omega}) = \frac{1}{|B_p(e^{j\omega})|^2} = \frac{1}{|1+\sum_{k=1}^{p}\beta_k e^{-j\omega k}|^2}$

Frugal Sensing

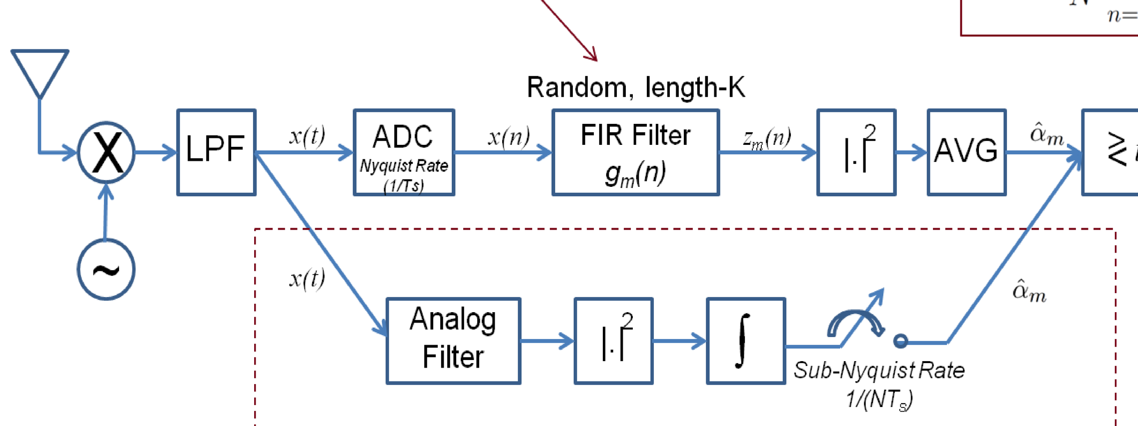


Estimate the AR spectrum from few bits

Complex PN - known at the FC

$$g_m(n) = \begin{cases} (1/\sqrt{2K})(\pm 1 \pm j) & \text{if} \quad 0 \leq n \leq K-1 \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_m := \mathbb{E}[|z_m(n)|^2]$$
$$\hat{\alpha}_m = \frac{1}{N}\sum_{n=0}^{N-1}|z_m(n)|^2$$



Equivalent analog measurement

AR Signal Model

$$\underbrace{x(n) + \sum_{k=1}^{p}\beta_k x(n-k)}_{\beta_k:\ \text{AR parameters}} = \underbrace{v(n)}_{\text{Complex WGN}}$$

Signal Autocorrelation

$$r_x(l) + \sum_{k=1}^{p}\beta_k r_x(l-k) = \delta(l), \ \forall\, l \in \mathbb{Z}^+$$

Power Measurement

$$\begin{aligned} \alpha_m &= \mathbf{g}_m^H\mathbf{R}_x\mathbf{g}_m = \mathbf{g}_m^H\left(\sum_{k=-(K-1)}^{K-1} r_x(k)\boldsymbol{\Theta}_k^K\right)\mathbf{g}_m \\ &= \sum_{k=-(K-1)}^{K-1} \underbrace{\mathbf{g}_m^H\boldsymbol{\Theta}_k^K\mathbf{g}_m}_{q_m(k)} r_x(k) \\ &= \mathbf{q}_m^T\mathbf{r}_x \end{aligned}$$

$$\mathbf{q}_m := [q_m(0), 2\,\mathrm{Re}\{q_m(1)\}, \cdots, 2\,\mathrm{Re}\{q_m(K-1)\}, 2\,\mathrm{Im}\{q_m(1)\}, \cdots, 2\,\mathrm{Im}\{q_m(K-1)\}]^T \in \mathbb{R}^{2K-1}$$
$$\mathbf{r}_x := [r_x(0), 2\,\mathrm{Re}\{r_x(1)\}, \cdots, 2\,\mathrm{Re}\{r_x(K-1)\}, 2\,\mathrm{Im}\{r_x(1)\}, \cdots, 2\,\mathrm{Im}\{r_x(K-1)\}]^T \in \mathbb{R}^{2K-1}$$

1-Bit Measurement

$$b_m = \mathrm{sign}(\mathbf{q}_m^T\mathbf{r}_x - t_m)$$

Goal: Estimate AR parameters from $\{b_m\}_{m=1}^M$

Alternating Minimization

Initialization:

- LP feasibility problem:

$$\begin{aligned} \text{find} \quad & \mathbf{r}_x \\ \text{s.t.} \quad & b_m(\mathbf{q}_m^T\mathbf{r}_x - t_m) \geq 0, \ \forall\, m \in \{1, \cdots, M\} \\ & \mathbf{F}\mathbf{r}_x \geq \mathbf{0}, \\ & \mathbf{r}_x \in \mathcal{P} \end{aligned}$$

AR parameter update:

- Least squares: $\boxed{\boldsymbol{\beta} = -(\bar{\mathbf{R}}_x^H\bar{\mathbf{R}}_x + \epsilon\mathbf{I}_K)^{-1}(\bar{\mathbf{R}}_x^H\tilde{\boldsymbol{\rho}})}$

Autocorrelation vector update

- QP problem:

$$\begin{aligned} \min_{\mathbf{r}_x} \quad & \|\boldsymbol{\Pi}\mathbf{r}_x - \mathbf{e}_1^K\|_2^2 \\ \text{s.t.} \quad & b_m(\mathbf{q}_m^T\mathbf{r}_x - t_m) \geq 0, \ \forall\, m \in \{1, \cdots, M\} \\ & \mathbf{F}\mathbf{r}_x \geq \mathbf{0}, \\ & \mathbf{r}_x \in \mathcal{P} \end{aligned}$$

Algorithm 1 : AM for AR PS Estimation

Initialization: Solve LP to obtain $\mathbf{r}_x^{(0)}$. Set $k := 0$.

Repeat

- Fix $\mathbf{r}_x^{(k)}$. Update $\boldsymbol{\beta}^{(k+1)}$ according via Least Squares.
- Fix $\boldsymbol{\beta}^{(k+1)}$. Update $\mathbf{r}_x^{(k+1)}$ by solving the QP problem.
- Compute cost value $v^{(k+1)} = \|\tilde{\mathbf{R}}_x^{(k+1)}\tilde{\boldsymbol{\beta}}^{(k+1)} - \mathbf{e}_1^K\|_2^2$
- Set $k := k + 1$.

Until Improvement in cost function $<$ *tolerance factor* in the last 10 iterations OR specified no. of iterations exceeded.

Features of AM algorithm

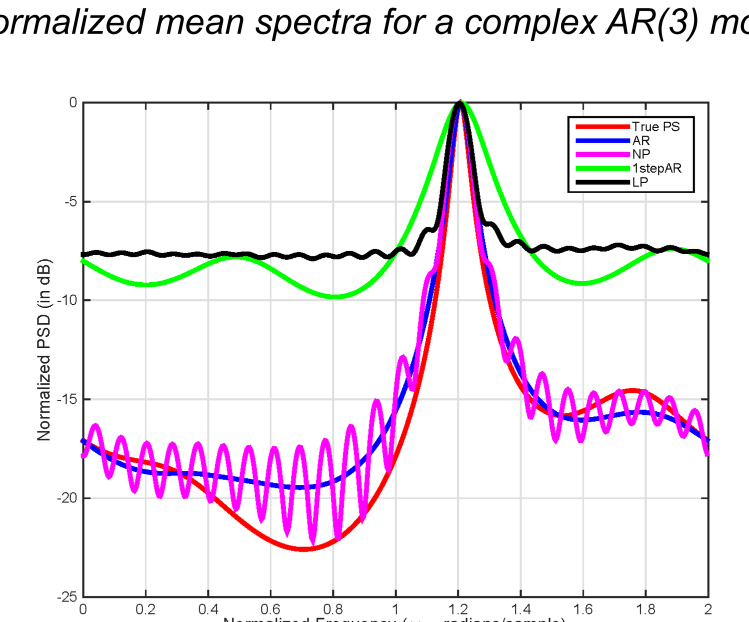
- Guaranteed convergence in terms of cost function
- Every limit point of the iterates is a stationary point (Ref: Chen *et al.*, "Maximum Block Improvement and Polynomial Optimization", *SIAM J. Optim.*, 2012)

Current State-of-Art

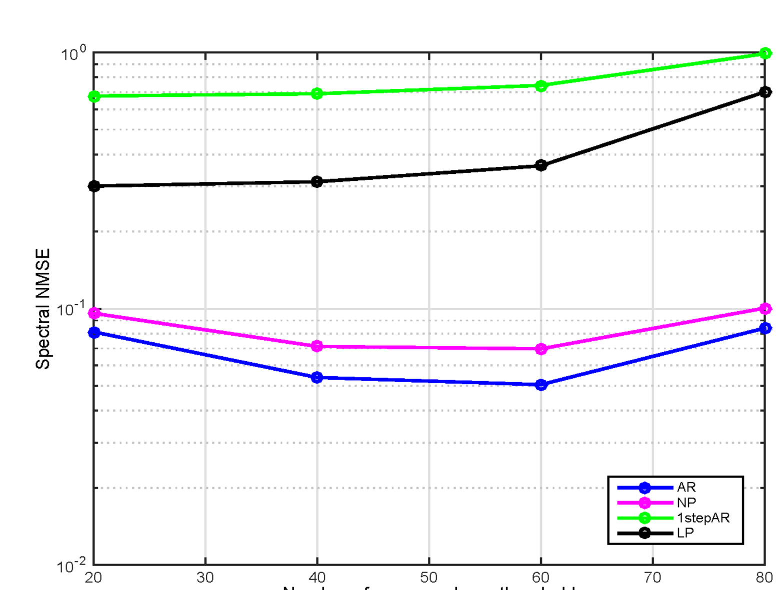
- Non-parametric LP approach (Ref: O. Mehanna and N. D. Sidiropoulos, "Frugal Sensing: Wideband Power Spectrum Sensing from few bits", *IEEE TSP*, May 2013)
- Classical AR spectrum estimation (Ref: P. Stoica and R. L. Moses, "Spectral Analysis of Signals") [Not applicable when the measurements are coarsely quantized]

Numerical Results

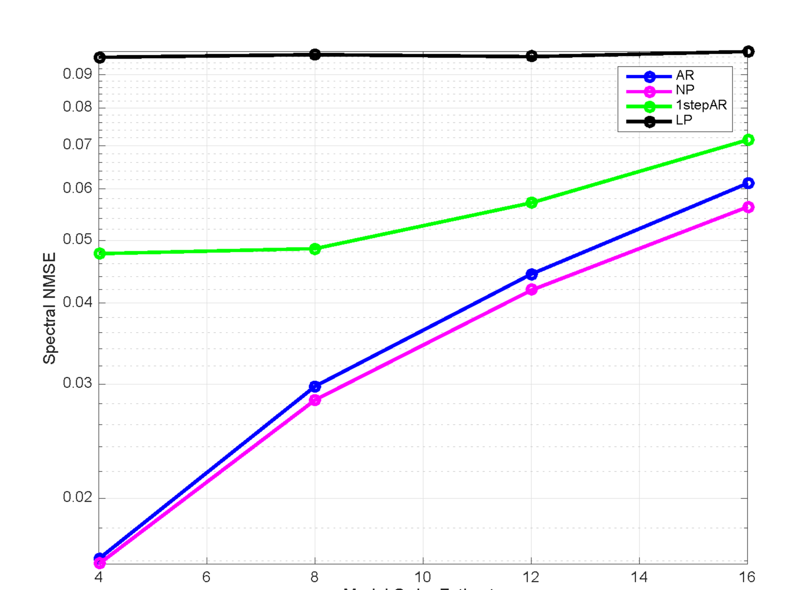
Normalized mean spectra for a complex AR(3) model



Spectral NMSE vs number of sensors above threshold for AR(5) models



Spectral NMSE vs model order over-estimate



Conclusions

- Adequate wideband AR spectrum estimation possible from few bits.
- Exploiting underlying AR structure produces more accurate estimation results as compared to non - parametric approaches when modeling assumption valid.